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EQUATIONS OF DISLOCATION PLASTICITY WITH LARGE DEFORMATIONS

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In investigations of the dynamic plasticity the simplest model of the theory of dislocation, including Orovan's equation

$$\dot{\gamma} = bNv, \tag{1a}$$

the law governing the motion of the dislocations

$$v = v(\tau) \tag{1b}$$

and the equation of kinetics of dislocations

$$N = N(\gamma), \tag{1c}$$

where γ is the shear deformation; τ, tangential stress; N, density of dislocations; v, their slipping velocity; and b, absolute magnitude of Burgers vector, is often used. It is also assumed that all dislocations are mobile and slip with identical velocities. Equation (1c) is usually written in the form $N = N_0 + A\gamma^s$, where s is a quantity of the order of one, and the motion is described either by the law of viscous friction

$$\tau b = Bv, \tag{2a}$$

or by Taylor's empirical formula [1]

$$v = v_0 \exp(-\tau_0/\tau) \tag{2b}$$

(B is the coefficient of viscous friction).

This very simple model corresponds to conditions of superbarrier slipping with uniform chaotic distribution of dislocations and can be used to describe small deformations of metals with low initial dislocation density. However, it neglects the fact that the uniform dis-

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tribution rapidly becomes unstable and already for relatively small deformations dislocations become organized in some nonuniform structures (slipping bands, clumps, plates, cells, etc.), after which the nature of the motion of the dislocations changes markedly and the assumption that all dislocations move with the same velocity is no longer applicable. In most metals, especially in bcc metals (aluminum, copper, nickel), a cellular structure forms rapidly. With subsequent deformation, the evolution of the cellular structure proceeds in two stages. In the first stage the size of the cell λ decreases with increasing N in accordance with Holt's empirical formula

$$\lambda(N) = K/\sqrt{N}, \tag{3}$$

where K = 15...20 [2]. At the second stage the size of the cells remains constant.

Plastic deformation is accompanied by multiplication of dislocations and an increase of N, as a result of which the metal hardens. Both for the uniform distribution and for cellular structures, deformation hardening is described by the approximate dependence

$$Y = Y_0 + \alpha Gb\sqrt{N}, \tag{4}$$

where Y is the yield point with respect to shear; G is the shear modulus. The magnitudes of the coefficients α differ somewhat for the uniform distribution and for the cellular structure and, moreover, vary little from metal to metal, but in the first approximation we can set $\alpha = 0.5$. For $\tau < Y$, almost all dislocations are firmly confined by the walls of the cell. For $\tau > Y$, separate dislocations break away from the walls and move into the interior of the cells until they collide with the walls. Usually, τ is only slightly greater than Y, even with high-velocity deformation, so that the cellular framework remains stable. The simplest kinetic equation for this case can be written in the form

$$dN/dt = m\dot{\gamma}/b\lambda - \dot{\gamma}Nh/b, \tag{5}$$

where λ/m is the average kinetic transit distance of the dislocations; h is the effective annihilation cross section for collisions of dislocations with opposite signs. Integration of (5), using (3), gives

$$\sqrt{N} = (m/Kh)[1 - \exp(-\gamma h/2b)]. \tag{6}$$

The first stage is completed for some γ_1 and $\sqrt{N_1} = K/\lambda_1$. For $\gamma > \gamma_1$, Eq. (5) must be integrated with constant λ, which leads to

$$N = m/h\lambda_1 + (N_1 - m/h\lambda_1)\exp[-(\gamma - \gamma_1)h/b]. \tag{7}$$

From the experimental data for cellular structures in copper, presented in [3], we can conclude that m = 1.25 and h = 0.6b. Such a small magnitude of the effective h must be explained. The cross section for annihilation for a uniform distribution is about 10b [4], but for nonuniform structures with separated dislocations of opposite signs h must be set equal to twice the product of this quantity by the probability of an encounter between dislocations of opposite sign accompanying trapping by a wall. For copper this probability equals 0.03, which indicates the quite high degree of ordering of the separation and correlation of the positions of sources and sinks of dislocations in the cellular structure. The quantity h can vary during the course of deformation, if the deformation is accompanied by an appreciable change in the ordering of the walls of the cells.

Relations (6) and (7) in principle solve the problem of the specific form of the kinetic equation (1c) for the cellular structure. The difficulty lies in the fact that, at the present time, there is not yet enough data on the specific values of the points at which the regime γ_1 changes for different metals and their dependence on the deformation regime. For example, for slow deformation of copper K = 16, λ_1 = 2 μm, and γ_1 = 0.05. But, for shock-wave hardening of copper, the cells in the copper are fragmented to sizes of 0.15 μm [5]. Judging from the strong dependence of the position of the transition on the temperature of deformation and the melting temperature of the metal, this phenomenon is determined by the thermally activated climbing of dislocations. If this is true, then under the conditions of high-velocity deformation, dislocation climbing can be neglected, using (6) up to deformations of, at least, the order of 0.2...0.4.

For the cellular structure, the density of mobile dislocations n differs from the average density N, so that (1a) should be used in the form $\dot{\gamma} = bnv$, and the problem of determining n thus arises. We shall use for this purpose the thermodynamic approach, assuming that from the possible regimes, which give a fixed velocity of deformation, a regime is selected which minimizes the resistance to deformation. The thermodynamic approach in this form was first used in [6], as a result of which it was possible, in particular, to explain qualitatively the empirical formula (3).

Thus we assume that for $\tau > Y$ the internal volumes of the cells are filled with mobile dislocations with density n. As a result, the contribution of the cellular framework, determined by the quantity $(N - n)$, to τ decreases, but a corresponding contribution of mobile dislocations and a viscous component appear. Mobile dislocations in interior volumes of the cells do not sustain the stress applied to the cellular framework, and for them the stress of self-blocking is determined only by the magnitude of n. For the law of motion (2a),

$$\tau = Y_0 + \alpha Gb(\sqrt{N-n} + \sqrt{n}) + \dot{\gamma}B/nb^2. \tag{8}$$

Applying to (8) the condition

$$d\tau/dn\,|_{\dot{\gamma},N=\text{const}} = 0,$$

we obtain

$$\frac{\alpha Gb}{2}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{N-n}}\right) - \frac{\dot{\gamma}B}{n^2b^2} = 0. \tag{9a}$$

Analogously, for the law of motion (2b),

$$\frac{\alpha Gb}{2}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{N-n}}\right) + \frac{nY_0}{\ln^2\left(\dot{\gamma}/bnv_0\right)} = 0. \tag{9b}$$

Thus, with deformation in the cellular regime, system (1) must be replaced by the kinetic equation (6) [or (7)] and Eqs. (8) and (9), for which, by eliminating n we determine the value of $\dot{\gamma}$ corresponding to the instantaneous values of τ, N. The use of Eq. (9) in numerical investigations does not present any special difficulties. These equations are applicable until n exceeds some critical fraction N, i.e., as long as the cellular structure is stable. In particular, for $n \ll N$ we obtain the estimate

$$n \simeq (2\dot{\gamma}B/\alpha Gb^3)^{2/3}.$$

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